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STATISTICAL ANALYSIS OF THE LMS (LAST MEAN SQUARES) AND
MODIFIED STOCHAST (U) CALIFORNIA UNIV IRVINE DEPT OF
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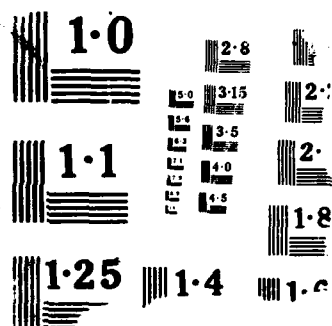
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Research work on the stochastic behavior of the last mean squares (LMS) and related algorithms has yielded results in four major areas. The transient mean performance of an analysis supported by simulations. Significant progress has been made in determining the joint transient and steady state probability density functions of the time domain LMS weight error gaussian jammers. The effects of non-linearities on the time domain LMS algorithm have been analyzed.

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I. In the second year (March 15, 1987 - April 15, 1988) of AFOSR support, research work on the stochastic behavior of the LMS and related algorithms has yielded results in four major areas:

A. Data Normalized LMS-Transient behavior [5]

This work generalized the efforts of the previous year to the transient behavior of arbitrary linear filters for estimating the input power level. The transient mean performance of an exponentially weighted power estimator was analyzed and the analysis supported by simulations.

B. Joint Probability Density Functions of the LMS Weights during Adaptation [2.6.11]

Significant progress has been made in determining the joint transient and steady-state probability density functions of the time domain LMS weights [6] and the complex scalar frequency domain weights [2,11]. It was shown in [6] that, after an initial transient, the weights are jointly gaussian for small values of μ with time-varying mean vector and covariance matrix given by the solution of the well-known difference equations for the weight vector mean and covariance matrix. For the complex scalar LMS algorithm, it was shown in [2,11] that the real and imaginary parts of the fluctuations about the Wiener weight are jointly log-normal distributed for small n and statistically dependent gaussian random variables for large n .

C. Application of the Results of B to a Spread Spectrum Communications Problem [4.7]

The effect of the weight fluctuations of the ALE on the bit error rate of a Direct-Sequence Spread Spectrum Communication system has been investigated [4,7]. The ALE is inserted in the system in order to reject narrowband interference. The converged ALE weights are modeled as the parallel connection of a deterministic FIR filter (Wiener weights) and a random FIR filter. The output of the random FIR filter was shown to be non-gaussian and to significantly degrade the error probability. Error probability expressions were derived for the bit error rate for monochromatic and narrowband sinusoidal gaussian jammers [4].

D. Non-Linear Effects in LMS Adaptation [1.3.8-13]

The effects of the various non-linearities on the behavior of the time domain LMS algorithm [1,3, 8-13] have been successfully analyzed:

1) It was shown [1] that the stopping phenomena in quantized LMS adaptation can be removed by the addition of spectrally shaped dither without significantly effecting the steady-state mean square error of the quantized algorithm.

2) It was shown [3] that a saturation type error non-linearity in the weight up-date equation causes a slow-down in the algorithm convergence rate directly proportional to the degree of saturation. Quoting from page 23 of [3]: "Thus, the trade-off between speed of convergence and saturation was investigated for a fixed steady-state excess mean-square-error. A functional relationship was derived between the convergence rate of the LMS algorithm and the number of bits in a fixed point implementation of the feedback error signal. This relationship was tabulated and shows that 1) starting with a sign detector, as one increases the number of bits in the error signal representation, the convergence rate is increased by nearly a factor of two for each additional bit, 2) as the number of bits is increased further, additional bits buy very little in additional convergence speed, and 3) as the number of bits increase further, the behavior approaches that of the linear algorithm".

3) Analog implementation of the LMS and Block LMS algorithms have been shown equivalent with respect to adaptation speed and steady-state mis-adjustment errors. However, analysis and simulation of digital implementations of the same algorithms have shown significantly different behavior with respect to transient response and algorithm stalling [8,12] for a fixed number bits. Mathematical models have been derived which predict the different behaviors and allow one to design the appropriate algorithms.

4) The mathematical models derived in [3] have been extended to the non-white data case for the LMS algorithm configured as an Adaptive Line Enhancer [9,14]. Algorithm convergence slowdown is predicted by the theory. Weak signal suppression is also predicted by the theory. Both predictions are supported by simulations.

5) The nonlinear error feedback effects analyzed in [3] have been extended to the entire LMS weight update term. A $[1-\exp(-x)]$ nonlinearity is used to model the effects of saturation [10,13]. The tradeoff between extent of saturation, algorithm convergence speed and steady-state excess mean square error are studied as in [3]. By comparison with [3], it is concluded that "there is no significant difference in the behavior of digital implementations of the LMS algorithm whether round-off occurs before or after multiplying the error by the data".

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These results impact significantly on the usefulness and applications of LMS adaptive filtering:

A. Although the LMS algorithm is simple to implement, one of its significant drawbacks is speed of convergence. The work in [1] enables one to design the algorithm so as to obtain a fast response to changing input power levels with sufficient smoothing for fixed unknown power levels.

B. Although the mean square error behavior of the LMS algorithm is well understood, there are many situations where additional statistical information about the weight vectors would be useful. These cases include: 1) detection of narrowband line component in background noise using the weight vector as a test statistic (adaptive line enhancer) [6], 2) using the adaptive filter output as a test statistic, 3) time delay estimation using the weights [6].

C. Recently the LMS algorithm has been imbedded in larger systems for the purposes of enhancing desired signals and canceling undesired signals. In this mode of operation, it is often necessary to know more than just second moment statistics of the weights. As an example, the LMS canceller has been used to cancel undesired narrowband interference in spread-spectrum communication systems. [4] makes use of the results in [6] to study the bit error probability of a DS Spread-Spectrum system using an ALE for narrowband interference rejection. The effects of the ALE weight misadjustment errors on the bit error rate are investigated. The converged ALE weights are modeled as a parallel connection of a deterministic FIR (the Wiener weights) and a random FIR filter (weight mis-adjustment errors) with gaussian statistics. Using properties of the ALE weight mis-adjustment errors derived elsewhere and in [6], error probability expressions are derived for the bit error rate for monochromatic and sinusoidal jammers.

D. Digital implementation of the LMS algorithm essentially involves some non-linear modifications of the basic algorithm. Understanding the effects of various non-linearities is important in choosing the kind of implementable non-linearities that yield rapid convergence rates and small steady-state mis-adjustment errors. The theory developed in [1,3,9,10,13,14] enables one to better understand this non-linear behavior.

When the LMS algorithm is implemented in block form to take advantage of the speed of block processing, finite word effects are different for the LMS and BLMS algorithms. Using gaussian data models, [8,12] investigated these differences and showed that the LMS algorithm is less sensitive to finite word effects than the Block LMS algorithm.

II. List of Publications

A. Journal Articles

1. "Quantization Effects in the Complex LMS Algorithm-Linearization Using Dither -- Applications," coauthor, IEEE Trans. on Circuits and Systems, Vol. CAS-35, No. 4, April 1988.
2. "Further Results on the Joint Characteristic Function of the Complex Scalar LMS Adaptive Weight," IEEE Trans. on Circuits and Systems, Vol. CAS-35, No. 4, April 1988.
3. "On Error Saturation Nonlinearities in LMS Adaptation," IEEE Trans. on Acoustics, Speech and Signal Processing, Vol ASSP-36, No. 4, April 1988.
4. "Error Probabilities for DS Spread-Spectrum Systems Using an ALE for Narrow-Band Interference Rejection," IEEE Trans. on Communications, Vol. COM-36, No. 5, May 1988.
5. "A Weighted Normalized Frequency Domain LMS Adaptive Algorithm," IEEE Trans. on Acoustics, Speech and Signal Processing Vol. ASSP-36, No. 7, July 1988.
6. "On the Probability Density Function of the LMS Adaptive Filter Weights," accepted IEEE Trans. on Acoustics, Speech and Signal Processing, March, 1988.

B. Conference Papers

7. "Error Probabilities in Spread-Spectrum Systems Using an ALE for Narrowband Interference Rejection," 2nd Annual Pacific Rim Conference on Communications, Computers and Signal Processing, Victoria, B.C., Canada, Paper No. 19-3, June 1987.
8. "Nonlinear Quantization Effects in the LMS and Block LMS Adaptation Algorithms-A Comparison", Proceedings of IEEE International Conf. on Acoustics, Speech and Signal Processing, Paper No. 25.D4.4, April 1988 New York, N. Y.

9. "Saturation Effects in the LMS Adaptive Line Enhancer-Response to Multiple Sinusoids", Proceedings of IEEE International Conf. on Acoustics, Speech and Signal Processing, Paper No. 35.D6.12, April 1988, New York, N.Y.

10. "Dynamic Range and Finite Wordlength Effects in Digital Implementations of the LMS Algorithm", Proceedings of IEEE International Symposium on Circuits and Systems, Invited Paper S130.6, June 1988, Espoo, Finland.

C. Papers submitted for Publication

11. "On an Approximate Solution of a Partial Differential-Difference Equation for the Joint Characteristic Function of the Complex Scalar LMS Adaptive Weight," submitted IEEE Trans. on ASSP, Feb. 1987, in revision Jan. 1988.

12. "Nonlinear Quantization Effects in the LMS and Block LMS Adaptive Algorithms-A Comparison", submitted to IEEE Trans. on ASSP, February, 1988.

13. "Dynamic Range and Finite Word Effects in Digital Implementation of the LMS Algorithm", submitted to IEEE Trans. on ASSP, March, 1988.

14. "Saturation Effects in the LMS Adaptive Line Enhancer-Transient Response" (co-author), submitted to IEEE Trans. on ASSP, April, 1988.

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